Quantum discord dynamical behaviors due to initial system-cavity correlations

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Abstract

We analyze the roles of initial correlations between the two-qubit system and a dissipative cavity on quantum discord dynamics of two qubits. Considering two initial system-cavity states, we show that the initial system-cavity correlations not only can initially increase the two-qubit quantum discord but also would lead to a larger long-time quantum discord asymptotic value. Moreover, quantum discord due to initial correlations is more robust than the case of the initial factorized state. Finally, we show the initial correlations' importance for dynamics behaviors of mutual information and classical correlation.

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Introduction: Recently, lots of interest have been devoted to the definition and understanding of correlations in the quantum systems. Entanglement is a kind of quantum correlation that has been playing a central role in quantum information and communication theory [1]. However there are other nonclassical correlations apart from entanglement [2-4] that can be of great importance to these fields. In order to characterize all nonclassical correlations, Ollivier and Zurek introduced what they called quantum discord. They marked the beginning of a new line of research shifting the attention from the entanglement vs. separability dichotomy to the quantum vs. classical paradigm. The quantum discord measures quantum correlations of a more general type than entanglement, there exists separable mixed states having nonzero discord [5]. Interestingly, it has been proven both theoretically and experimentally that such states provide computational speedup compared to classi-

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cal states in some quantum computation models [5,6].

Understanding of the quantum dynamics of open systems is a very important task in many areas of physics ranging from quantum optics to quantum information processing and to quantum cosmology. Therefore, the study of quantum and classical correlations' dynamical behaviors in the presence of both Markovian [7,8] and non-Markovian [9,10] decoherence has attracted much attention in recent years. It is believed that the quantum correlations measured by the quantum discord, in the Markovian case, decay exponentially in time and vanish only asymptotically [11], contrary to the entanglement dynamics where sudden death may occur [12]. In particular, Refs. [13,14] have discovered that quantum discord can be completely unaffected by Markovian depolarizing channels or non-Markovian depolarizing channels for long intervals of time, and this phenomenon has been observed experimentally [15]. In these above studies, quantum discord dynamics of open system in which system and environment are initially separable has been analyzed in detail. As everyone knows it is possible, and often unavoidable in experiment, to create correlations between system and environment, therefore such correlations will play important roles in the time evolution of the system. So the study of quantum correlation dynamics due to the initial system-environment correlations is certainly necessary.

The influence of initial correlations on the open system dynamics has recently been intensively studied [16-22]. In order to clear what happens to the quantum discord in this situation, we study an exactly solvable model for the time evolution of two atoms interacting with a lossy cavity. In this model where the coupling strength effects are considerable, the presence of system-cavity correlations invalidates the initial state in which system and cavity are independent. We investigate quantum discord dynamics for two initial states of two atoms and the lossy cavity, in which the initial reduced density matrixes of the atomic system and the cavity are the same. Under a certain condition, the initial system-cavity correlations not only can initially increase the atomic quantum discord but also would lead to a larger long-time quantum discord asymptotic value. For another condition, correlations between two atoms and the lossy cavity, can more effectively restrain the reduction of quantum discord than the case of the factorized state. And then we also analyze the different dynamics behaviors of mutual information and classical correlation due to the initial system-cavity correlations. These findings obtained from our article show that, dynamics of mutual information, quantum discord and classical correlation not only depend on the system degrees of freedom, the initial system-cavity correlations must also be properly taken

into account.

Quantum discord and classical correlation: We now present a brief review of the classical correlation and quantum discord. In classical information theory, the information can be quantified by Shannon entropy $H(X) = -\sum_x P_{|X=x}\log P_{|X=x}$, where $P_{|X=x}$ is the probability with X being x. Similarly, the joint entropy, which measures the total uncertainty of a pair of random variables X and Y, is defined as $H(X,Y) = -\sum_{x,y} P_{|X=x,Y=y}\log P_{|X=x,Y=y}$, with $P_{|X=x,Y=y}$ being the probability in the case of $X = x$ and $Y = y$. Then the total correlation between X and Y can be measured by the mutual information which is defined as $I(X:Y) = H(X) + H(Y) - H(X,Y)$, whose quantum version can be written as [23]

$$\mathcal{I}(X:Y) = S(\rho_X) + S(\rho_Y) - S(\rho_{XY}), \quad (1)$$

where $S(\rho) = -Tr(\rho\log\rho)$ is the von Neumann entropy of ρ, and $\rho_X(\rho_Y)$ is the reduced density matrix of ρ_{XY} by tracing out $Y(X)$. By introducing the conditional entropy $H(X|Y) = H(X,Y) - H(Y)$, we can rewrite the mutual information as

$$I(X:Y) = H(X) - H(X|Y), \quad (2)$$

where $H(X|Y) = \sum_y p_{Y=y} H(X|Y=y) = -\sum_{x,y} p_{X=x,Y=y}\log p_{X=x,Y=y}$ is the conditional entropy of the random variables X and Y for the average uncertainty about the value of X given that the value of Y is known. In order to generalize the above equation to the quantum domain, we measure the subsystem Y by a complete set of projectors $\prod_i$, corresponding to the outcome i, which yields $\rho_{X|i} = Tr_Y(\prod_i \rho_{XY}\prod_i)/p_i$, with $p_i = Tr_{XY}(\prod_i \rho_{XY}\prod_i)$. Then the quantum mutual information can alternatively defined by

$$\mathcal{J}_{\prod_i}(X:Y) = S(\rho_X) - S_{\prod_i}(X|Y), \quad (3)$$

where $S_{\prod_i}(X|Y) = \sum_i p_i S(\rho_{X|i})$ is conditional entropy of the quantum state. The above quantity strongly depends on the choice of the measurements $\{\prod_i\}$. By maximizing $\mathcal{J}_{\prod_i}(X:Y)$ over all $\{\prod_i\}$, we define the classical correlation between X and Y

$$\mathcal{C}(X:Y) = \max_{\prod_i} J_{\prod_i}(X:Y), \quad (4)$$

and the quantum discord as

$$\mathcal{D}(X:Y) = \mathcal{I}(X:Y) - \mathcal{C}(X:Y), \quad (5)$$

which is interpreted as a measure of the quantum correlation [2-4]. It is zero only for states with classical correlations and nonzero for states with quantum correlations. In particular, quantum discord is equal to the entanglement of formation for pure states, it is not true for mixed states, since some states present finite quantum discord even without entanglement [2].

The model: We consider two atoms A and B interacting with a dissipative cavity. The Hamiltonian of such total system in the rotating-wave approximation is given by $H = H_0 + H_{int}$, which, in the basis $\{|gg\rangle, |eg\rangle, |ge\rangle, |ee\rangle\}$ reads

$$H_0 = \omega_0(\sigma_+^A\sigma_-^A + \sigma_+^B\sigma_-^B) + \omega_c a^\dagger a, \quad (6)$$

$$H_{int} = \Omega(\sigma_+^A a + \sigma_+^B a) + h.c., \quad (7)$$

here $\sigma_\pm^A$, $\sigma_\pm^B$ are, respectively, the Pauli raising and lowering operators for atoms A and B, ω_0 is the Bohr frequency of two atoms, a and $a^\dagger$ are the annihilation and creation operators for the cavity mode, which is characterized by the frequency ω_c and the coupling constant Ω. For the sake of simplicity, in the following we assume that the two atoms interact resonantly with the dissipative cavity mode.

In this section, we investigate the dynamics of the two atoms interacting with a dissipative cavity by making use of the master equation. Firstly, we focus on the case in which the total system contains only one excitation. In the interaction picture, the exact dynamics of the two atoms is contained in the following master equation

$$\frac{d\rho}{dt} = -i[H_{int}, \rho] - \frac{\Gamma}{2}[a^\dagger a\rho - 2a\rho a^\dagger + \rho a^\dagger a], \quad (8)$$

where ρ is the density operator for the two atoms and the cavity mode, Γ is the decay rate of the cavity mode. In order to find the dynamics of two atoms, we solve the master equation in Eq. (8). In the case which the total system contains only one excitation, we need to solve a set of 16 differential equations obtained from Eq. (8). Then, tracing out the cavity degree of freedom, we obtain the reduced density matrix of the atomic system for the total system which only contains one excitation.

Then we treat another case that the total system contains at most two excitations. In this case the dynamics of two atoms can be effectively described by a four-state system in which three states are coupled to the cavity mode in a ladder configuration, and one state is completely decoupled from the other states and form the field. In the basis $\{|0\rangle = |gg\rangle, |+\rangle = (|eg\rangle + |ge\rangle)/\sqrt{2}, |-\rangle = (|eg\rangle - |ge\rangle)/\sqrt{2}, |2\rangle = |ee\rangle\}$, the Hamiltonians (6) and (7) can be rewritten

$$H_0' = 2\omega_0|2\rangle\langle 2| + \omega_0(|+\rangle\langle +| + |-\rangle\langle -|) + \sum_k \omega_k a_k^\dagger a_k, \quad (9)$$

$$H_{int}' = \sqrt{2}\Omega(|+\rangle\langle 0|a + |2\rangle\langle +|a) + h.c.. \quad (10)$$

From the total Hamiltonian given by (9) and (10), the subradiant state $|-\rangle$ does not decay, and the superradiant state is coupled to states $|0\rangle$ and $|2\rangle$ via the cavity mode. The transitions $|0\rangle \rightarrow |+\rangle$ and $|+\rangle \rightarrow |2\rangle$ have the same frequencies and identically coupled with the cavity. In the interaction picture, the dynamics of two atoms interacting with a lossy cavity can be treated in the following master equation

$$\frac{d\rho}{dt} = -i[H_{int}', \rho] - \frac{\Gamma}{2}[a^\dagger a\rho - 2a\rho a^\dagger + \rho a^\dagger a]. \quad (11)$$

Focusing on the case in which there are at most two excitations in the total system, we should solve a set of 64 differential equations obtained from Eq. (12). The symmetry properties of the system allow to transform the set of differential equations into decoupled subset of differential equations of smaller size. Finally, we obtain the density matrix of the reduced atomic system by

tracing out the cavity degree of freedom.

To illustrate the roles of the initial correlations between two atoms and cavity on the quantum discord dynamics of two atoms, we consider a initial condition $\rho_{ABc}^{(1)} = |\Psi\rangle_{ABc}\langle\Psi|$, with $|\Psi\rangle_{ABc} = \alpha|ge0\rangle + \beta|eg0\rangle + \gamma|gg1\rangle$ (having correlations), and here $|\alpha|^2 + |\beta|^2 + |\gamma|^2 = 1$ ($\alpha, \beta, \gamma \neq 0$), $|e\rangle$ and $|g\rangle$ are the exited state and ground state of atoms, $|0\rangle$ and $|1\rangle$ are the vacuum state and the single-photon state of the lossy cavity. Obviously, two atoms and the cavity in this initial state is

$$\rho_{AB} = \begin{pmatrix} |\gamma|^2 & 0 & 0 & 0 \\ 0 & |\alpha|^2 & \alpha\beta^* & 0 \\ 0 & \alpha^*\beta & |\beta|^2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \qquad (12)$$

$$\rho_c = \begin{pmatrix} |\alpha|^2 + |\beta|^2 & 0 \\ 0 & |\gamma|^2 \end{pmatrix}. \qquad (13)$$

Therefore, the initial correlations must illustrate clear effects on the dynamics of the atomic quantum discord. In the following, by comparing to the second initial condition that the two-atom system and the cavity are in the factorized state $\rho_{ABc}^{(2)} = \rho_{AB} \otimes \rho_c$, we mainly study the different atomic quantum discord dynamical behaviors due to these two initial atoms-cavity states, which contain the identical states of subsystems.

For $\rho_{ABc}^{(1)}$, it is composed by the basis $|ge0\rangle$, $|eg0\rangle$ and $|gg1\rangle$, that is to say the initial correlation state $\rho_{ABc}^{(1)}$ satisfies the first case in which the total system contains only one excitation. So the density matrix of the total system at time t can be calculated by Eq. (8). However, not only the basis ($|ge0\rangle$, $|eg0\rangle$ and $|gg1\rangle$) but also those basis ($|ge1\rangle$ and $|eg1\rangle$) are contained in the factorized state, so $\rho_{ABc}^{(2)}$ actually consists of two parts, one satisfying one excitation, and the other meeting two excitations. Hence the evolutional density matrix of $\rho_{ABc}^{(2)}$ can be acquired by Eqs. (8) and (12). Tracing out the cavity degree of freedom, we obtain the density matrix of the reduced atomic system for these two different initial states. In the basis $\{|gg\rangle, |eg\rangle, |ge\rangle, |ee\rangle\}$, we measure the atom B from the matrix $\rho_{AB}(t)$ by projecting on $\{\cos\vartheta|e\rangle_B + e^{i\phi}\sin\vartheta|g\rangle_B, e^{-i\phi}\sin\vartheta|e\rangle_B - \cos\vartheta|g\rangle_B\}$. Then the quantum discord and classical correlation could be calculated numerically using Eqs. (4) and (5).

Numerical results and discussions: In this article we study a system whose dynamics is described by the well-known damped Tavis-Cummings model. To see the effects of initial system-cavity correlations explicitly, We consider two distinct initial states, the system-cavity correlated state $\rho_{ABc}^{(1)}$ and the factorized state $\rho_{ABc}^{(2)}$. The reduced density matrices for both the atomic system and the lossy cavity are the same. For simplicity, we choose the parameters $\alpha = \sin\theta\cos\varphi$, $\beta = \sin\theta\sin\varphi$ and $\gamma = \cos\theta$ ($\theta \in [0, \pi/2]$ and $\varphi \in [0, \pi]$). In Fig.1, by choosing $\rho_{ABc}^{(1)}$ and

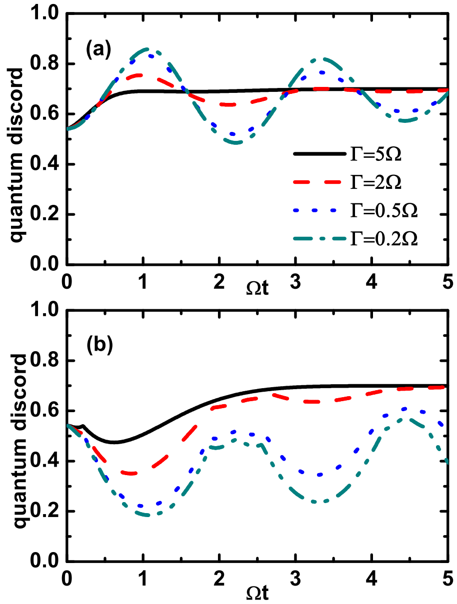

FIG. 1. Time evolution of quantum discord of two atoms as a function of the dimensionless quantity Ωt with $\theta = \pi/3$ and $\varphi = 3\pi/4$ (a) the initial system-cavity correlated state $\rho^{(1)}_{ABc}$ as the total initial state, (b) the factorized state $\rho^{(2)}_{ABc}$ as the total initial state.

$\rho^{(2)}_{ABc}$ as the initial states of the total system, we plot the time evolution of quantum discord for two atoms as a function of the dimensionless quantity Ωt, with $\theta = \pi/3$ and $\varphi = 3\pi/4$. We can observe that the atomic quantum discord can be initially increased to a maximum in the case $\rho^{(1)}_{ABc}$ (as shown in Fig.1(a)), which contains system-cavity correlations at $t = 0$. While in the factorized initial state $\rho^{(2)}_{ABc}$, the initially decreased atomic quantum discord to a minimum would happen (as shown in Fig.1(b)). Hereafter, how much the coupling strength of our regime influences the quantum discord? Since Γ is connected to the lossy cavity correlation time τ_B by the relation $\tau_B \approx 1/\Gamma$, and Ω is related to the time scale τ_R over which the state of the system changes, $\tau_R \approx 1/\Omega$. So $\Gamma/\Omega < 2$ characterizes the strong coupling regime, and $\Gamma/\Omega > 2$ means the weak coupling regime. Fig.1a presents the case for the initial system-cavity correlated state $\rho^{(1)}_{ABc}$, it can be seen that the initially increased quantum discord maximum increases with the decrease of the Γ/Ω corresponding to the enhancement of the coupling strength. We also clearly see that for the factorized initial state $\rho^{(2)}_{ABc}$ in Fig.1b, the initially decreased quantum discord minimum decreases with the decrease of the Γ/Ω.

Then, by investigating the subsystem of two atoms, we shall compare the roles of different initial system-cavity states ($\rho^{(1)}_{ABc}$ and $\rho^{(2)}_{ABc}$) on the atomic quantum discord dynamics. Although the initial reduced density matrices for both the atomic system and the lossy cavity are the same as in the full calculation, Fig.2 shows that the presence of the system-cavity correlations in the initial state changes the atomic quantum discord dynamics dramatically. In figs.2(a) and 2(b), quantum discord dynamics is given for the conditions $\theta = \pi/3$ and $\varphi = 3\pi/4$ in the strong coupling regime and weak coupling regime, respectively. For the case $\rho^{(1)}_{ABc}$, the atomic quantum discord can firstly increase to a maximum, then periodically decrease to a long-time asymptotic value in

the strong coupling regime. While in the weak coupling regime, quantum discord would reach a maximum at first and then gradually decrease to the asymptotic fixed value. For the other case $\rho^{(2)}_{ABc}$, quantum discord of two atoms initially reduce to a minimum, and eventually periodically increase to another long-time asymptotic value in the strong coupling regime (no oscillations are present in the weak coupling regime). Due to the subradiant state $|-\rangle$ in two initial states does not decay in the evolution process, the long-time asymptotic quantum discord would be acquired. The final long-time asymptotic value due to the system-cavity correlations is much larger than the case of the factorized state. So we can conclude that the initial system-cavity correlations not only can initially increase the atomic quantum discord but also would lead to a larger long-time quantum discord asymptotic value.

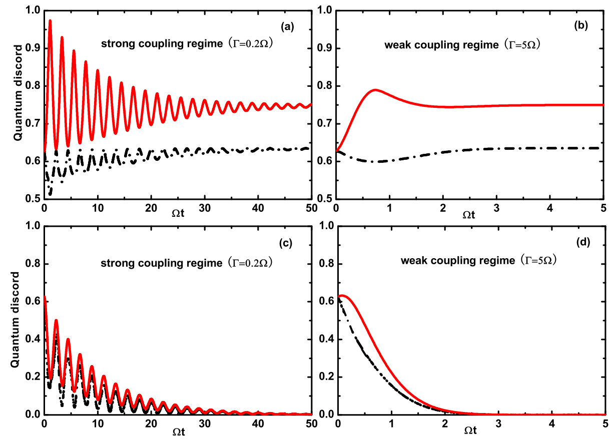

FIG. 2. Time evolution of quantum discord of two atoms as a function of the dimensionless quantity Ωt with $\theta = \pi/3$, (a) $\varphi = 3\pi/4$ and $\Gamma = 0.2\Omega$, (b) $\varphi = 3\pi/4$ and $\Gamma = 5\Omega$, (c) $\varphi = \pi/4$ and $\Gamma = 0.2\Omega$ and (d) $\varphi = \pi/4$ and $\Gamma = 5\Omega$. For the cases of (i) the initial system-cavity correlated state $\rho^{(1)}_{ABc}$ (red solid curve), (ii) the factorized state $\rho^{(2)}_{ABc}$ (dark dash-dotted curve).

Figs.2(c) and 2(d) give the atomic quantum discord dynamics for the conditions $\theta = \pi/3$ and $\varphi = \pi/4$ in the strong coupling regime and weak coupling regime, respectively. When $\varphi = \pi/4$, the initial atomic state does not contain the subradiant state $|-\rangle$, the asymptotic stationary state of two atoms is $\rho_{AB}(t \to \infty) = |gg\rangle\langle gg|$, so the long-time asymptotic value of the atomic quantum discord is zero. For both the initial system-cavity correlated state or the factorized state, in the strong coupling regime the atomic quantum discord presents damped oscillations while in the weak coupling regime quantum discord decays only asymptotically to zero. A comparison between the red solid curve and the dark dash-dotted curve in Figs.2(c) and 2(d) reveals that for the same initial atomic state, the atomic quantum discord dynamics due to the initial system-cavity correlations is more robust than the case of the factorized state. This suggests that correlations between two atoms and the lossy cavity, effectively restrains the reduction of atomic quantum discord.

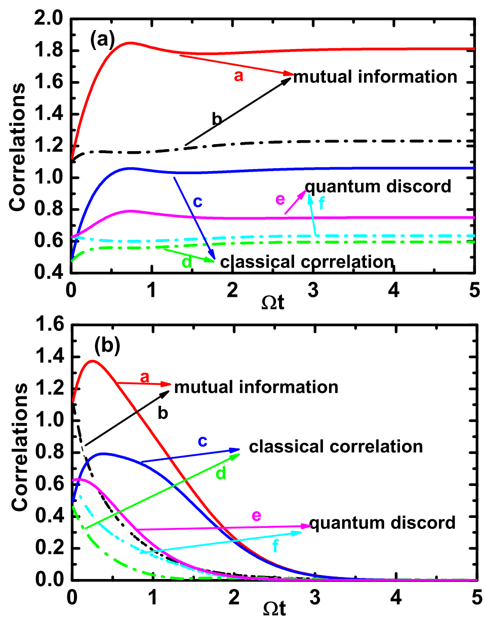

FIG. 3. Correlation (mutual information, quantum discord, classical correlation) dynamics of two atoms as a function of the dimensionless quantity Ωt in the weak coupling regime with $\theta = \pi/3$, (a) $\varphi = 3\pi/4$ and $\Gamma = 5\Omega$, (b) $\varphi = \pi/4$ and $\Gamma = 5\Omega$. For the cases of (i) the initial system-cavity correlated state $\rho^{(1)}_{ABc}$ (the solid curves a, c and e), (ii) the factorized state $\rho^{(2)}_{ABc}$ (the dash-dotted curves b, d and f).

To further understand the roles of different initial system-cavity states ($\rho^{(1)}_{ABc}$ and $\rho^{(2)}_{ABc}$) on dynamics of the total correlations, we study the atomic mutual information $\mathcal{I}$ and classical correlation $\mathcal{C}$, respectively. Fig.3 shows the time evolution of the atomic mutual information, quantum discord and classical correlation in the weak coupling regime. It is clearly found that for the initial correlated state $\rho^{(1)}_{ABc}$, the atomic classical correlation is greater than quantum discord except in the initial period of time. While the classical correlation is always less than quantum discord for the initial factorized state $\rho^{(2)}_{ABc}$. When $\varphi = 3\pi/4$, because of the initial system-cavity correlations, both the atomic classical correlation and mutual information can firstly increase to a maximum, and then get to a fixed long-time asymptotic value. In addition, the fixed long-time asymptotic values of classical correlation and mutual information due to $\rho^{(1)}_{ABc}$ are much larger than the case of $\rho^{(2)}_{ABc}$ (Fig.3(a)). This result is same as quantum discord dynamics. When $\varphi = \pi/4$, the atomic classical correlation and mutual information due to the correlated state $\rho^{(1)}_{ABc}$ would increase to a maximum value at first, then reduce asymptotically to zero. This finding is different from quantum discord dynamics, which only decays to zero (Fig.3(b)). Comparing to the case of the initial factorized state $\rho^{(2)}_{ABc}$, classical correlation and mutual information of two atoms also can be more robust.

Conclusion: In conclusion, we have studied quantum discord of two qubits in the presence of initial system-cavity correlations. The correlations between the qubits and a lossy cavity is found to have important effect on the time evolution of quantum discord when two qubits couple with a common lossy cavity. Finally, we also analyze the different dynamics behaviors

of mutual information and classical correlation due to the initial system-cavity correlations. In comparison with some recent work on the initial factorized state between the system and environment, our present work might be more practical to explain the total correlation dynamics behaviors of the system.

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